Comparative Study of 1D Codes for Site Response Analyses

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Abstract
During the last decades, significant number of study has been done on the effects of local soil conditions on ground motion. These effects are commonly evaluated by experimental and/or numerical methods. The numerical analyses are generally performed either in frequency or time domain. Time domain analyses usually incorporate in the equations of motion viscous damping which is generally assumed to be of the Rayleigh type, i.e. stiffness- and mass-proportional, or in a simplified form, only stiffness-proportional. Both formulations result in a frequency-dependent damping, in contrast to the soils experimental behavior. In this study, several linear analyses were conducted for an idealised soil deposit with two different time domain computer codes in order to explore the influence of Rayleigh damping formulation on site response analyses. Moreover, nonlinear analyses were performed for the selected Orvieto site in Central Italy. The results of the analyses show that the use of the simplified Rayleigh damping formulation can significantly underestimate the seismic response, especially for deep deposits and high frequency input motion. In addition, it is shown that with the use of full Rayleigh damping formulation, the results of analyses are more consistent with those obtained with a frequency-independent analysis.

Keywords: Site effects, numerical analyses, Rayleigh damping

1 INTRODUCTION
The changes in the intensity, frequency content and duration of the motion due to the propagation of the seismic waves in soil deposits and the presence of topographic features, are commonly referred as site effects. The mechanical properties of the soil deposit (i.e., shear modulus and damping ratio) greatly influence the propagation of seismic waves and therefore the characteristics of the surface ground motion. Moreover, the nonlinear behavior of soils during a seismic event has a predominant role in site response. Seed and Idriss (1969) introduced modern site response analyses techniques to understand nonlinear behavior of soils under earthquake loadings by using an equivalent linear scheme. However, there are cases (i.e. high seismic intensities at the rock base and/or high strain levels in the soil layers) in which the equivalent model cannot accurately represent the nonlinear behavior of the soil column. In these cases, a true nonlinear scheme should be used. A recent state-of-art about non linear analyses of site response can be found in Hashash et al. (2010). Since then several computer codes have been developed to analyze the problem of horizontally stratified layer overlying a homogeneous half-space and excited at the base by vertically propagating shear waves. Analyses are carried out in the frequency or time domain in order to solve the wave propagation equations. Different constitutive relations were developed to model the hysteretic behavior of soils under cyclic loading conditions. Equivalent linear SHAKE code (Schnabel et al. 1972) is the most commonly used frequency domain one-dimensional program. Few years later, bi-dimensional finite element programs QUAD-4 (Idriss et al., 1975) and QUAD4M (Hudson et al. 1994) have been developed which operate in time domain using an equivalent linear method. More recently, D-MOD2000 (Matasovic and Ordonez, 2012) and DEEPSOIL (Hashash, 2012) are widely used to perform one-dimensional nonlinear site response analyses. Specifically, D-MOD2000 works only in time domain while DEEPSOIL can perform 1D analysis both in frequency and time domains.

Time-domain nonlinear ground response analyses are seldom used in practice by non-experts users because parameter selection is often unclear and poorly documented. Kwok et al. (2007) conducted an extensive comparative study using five leading nonlinear site response codes in order to clarify two issues which were a source of confusion, that is
a) the specification of the input motion as “outcropping” (i.e. equivalent free surface motion) or “within” (i.e. motion occurring at depth within a site profile); b) specification of damping at small strains considering that most nonlinear codes utilize some form of viscous damping (Rayleigh damping) so that a finite level of damping is present regardless of the backbone curve. Regarding the second point, which is of interest for this paper, Kwok et al. (2007) compared results of linear time domain analyses for three site profiles to solutions from frequency domain analyses (taken as reference) and developed recommended procedures for the specification of Rayleigh damping which are hereafter summarized: a) full Rayleigh damping is preferred to simplified formulation which can produce either overdamping or underdamping; b) the target damping level should match the small-strain soil damping, c) as first approximation, the target frequencies (i.e. the frequencies for which the viscous damping produced by the model matches the target) could be selected as the first-mode site frequency and five times that frequency.

In this study, the specification of Rayleigh damping is investigated for two well-known time-domain codes D-MOD2000 and DEEPSOIL. Specifically, the influence of Rayleigh damping options (simplified or full formulation, control frequencies and target damping selection) is studied based on the results of a suite of linear and nonlinear comparative site response analyses. The frequency solutions from SHAKE analyses are taken as reference. In both D-MOD2000 and DEEPSOIL, viscous damping is introduced in terms of Rayleigh formulation to capture damping at very small strain. In particular, D-MOD2000 implements the simplified and full (with one or two control frequencies) Rayleigh damping formulation whereas the DEEPSOIL code also includes the extended Rayleigh damping formulation (with four control frequencies).

2 VISCOS DAMPING FORMULATIONS IN DYNAMIC ANALYSES

In time domain dynamic analyses the equations of motion are solved directly as a set of simultaneous equations. Such a solution would then require that the viscous damping matrix \([C]\) be known. The \([C]\) matrix is most commonly expressed by the full Rayleigh damping formulation (Chopra, 1995), i.e. a linear combination of both mass \([M]\) and stiffness \([K]\) matrices:

\[
[C] = \alpha [M] + \beta [K]
\]

where \(\alpha\) and \(\beta\) are the Rayleigh damping coefficients. It can be shown (Chopra, 1995) that the modal damping ratio \(\xi_j\) for the \(j\)th mode is:

\[
\xi_j = \frac{\alpha}{2\omega_j} + \frac{\beta\omega_j}{2}
\]

where \(\omega_j\) is the circular frequency for mode number \(j\). Determination of \(\alpha\) and \(\beta\) coefficients can be obtained with two approaches, the first uses a single control frequency and the second uses two control frequencies. In the single control frequency approach, \(\alpha\) and \(\beta\) are:

\[
\alpha = \frac{\xi_*}{\omega_*}, \quad \beta = \frac{\xi_*}{\omega_*}
\]

where \(\xi_*\) is the modal damping ratio at the control frequency \(\omega_*\). Usually, it is assumed \(\omega_* = \omega_1\) where \(\omega_1\) is the fundamental circular frequency of the soil deposit and \(\xi_*\) the target soil damping.

The relation between \(\xi_j\) and \(\omega_j\) computed from (2) is presented in Figure 1 (a) as bold line. This figure shows that \(\xi_j\) attains its minimum \(\xi_*\) at the control frequency \(\omega_* = \omega_1\).
while all other frequencies are more heavily damped. The stiffness-proportional damping is often used in dynamic analyses and corresponds to the limit case of the Rayleigh damping with $\alpha = 0$ in equation (1). In this case the coefficient $\beta$ is given by (Chopra, 1995):

$$\beta = \frac{2 \xi_j}{\omega_1}, \quad \xi_j = \frac{\omega_j}{\omega_1}$$

while the modal damping ratio $\xi_j$, according to (2) with $\alpha = 0$, linearly increases with frequency (Figure. 1c).

In the two control frequencies approach, assuming that the damping ratio attains the same value at the two modes $m$ and $n$, i.e. $\xi_m = \xi_n = \xi^*$, the expressions for $\alpha$ and $\beta$ are:

$$\alpha = \xi^* \frac{2 \omega_m \omega_n}{\omega_m + \omega_n}, \quad \beta = \xi^* \frac{2}{\omega_m + \omega_n}$$

The corresponding relationship between damping ratio $\xi_j$ and natural frequency $\omega_j$ is shown in Figure. 1b. It is observed that for the frequencies between $\omega_m$ and $\omega_n$, damping ratio is slightly less than the target value $x^*$ while for frequencies outside of this range larger damping ratios are obtained. The selection of modes $m$ and $n$ with specified damping ratio generally leads to damping ratios values reasonably close to the target value in all the modes contributing significantly to the response (Lanzo et al., 2003).

### 2.1 IMPLEMENTATION OF RAYLEIGH DAMPING IN THE PROGRAMS USED

To evaluate the effect of different Rayleigh damping options on seismic response analyses, time-domain computer codes, D-MOD2000 and DEEPSOIL were used. Table 1 shows different options adopted by the two codes for modeling viscous damping. Main differences regard the Rayleigh formulation (simplified, full or extended), the number of control frequencies (1, 2 or 4), and the way in which control frequencies are specified (automatically or by the user, layer by layer or for the whole soil deposit). It should be remarked that for all the codes the target damping specified by the user is a material damping ratio instead of the modal one. It is possible to demonstrate that a time-domain linear analysis assuming a material damping $\xi_{mat}$ constant along the profile is equivalent to a modal superposition analysis with modal damping ratio variable according to (2) with $a$ and $b$ computed by (3), (4) and (5) in which $\xi^* = \xi_{mat}$ (Chopra, 1995).

In the 1D lumped-mass nonlinear D-MOD2000 code the full Rayleigh damping formulation with two control frequencies is implemented. The user can specify the Rayleigh coefficients and the target damping ratio layer by layer. The viscous damping matrix $[C]$ is therefore assembled using the selected Rayleigh option (simplified or full). For the selection of control frequency the procedure implemented in QUAD4M was followed: the first one is the fundamental frequency of the whole system ($f_s$) while the second one is assumed $f_2 = nf_s$ with $n$ being the smallest odd integer so that $f_2$ is larger than the predominant frequency of the earthquake motion ($f_p$).
In DEEPSOIL target damping ratio can be specified layer by layer while the values of control frequencies must be selected by the user for the whole soil deposit. The extended Rayleigh formulation is also available in DEEPSOIL in addition to simplified and full ones. In this case the user must specify four control frequencies. The extended formulation is computationally time-consuming and therefore is seldom applied in practice.

3 LINEAR ANALYSES

Parametric 1D linear visco-elastic analyses were first carried out by using the time–domain codes D-MOD2000 and DEEPSOIL for six different accelerograms on an ideal soil profile. The results were compared with the assumed reference results obtained from PROSHAKE (frequency independent damping). The 1D profile used for the analyses (Figure 2) is characterized by thickness $H = 30m$, unit weight $\gamma = 20 kN/m^3$, shear wave velocity $V_s = 100 m/s$ and damping ratio $D = 5\%$. The fundamental frequency of soil deposit is therefore $f_s = 0.83Hz$ ($T_s = 1.2s$). The underlying bedrock is characterized by $\gamma = 22 kN/m^3$ and $V_s = 1000 m/s$. These six accelerograms employed in the analyses were selected in such a way that they cover a large frequency range (Table 2): the predominant frequency ($f_p$) of input motions lies between $0.9 Hz$ (close to $f_s$) and $8.3 Hz$ thus exciting higher modes of vibration of soil deposit. The acceleration response spectra (5% structural damping) of all selected time histories are shown in Figure 3.

The linear analyses with the full Rayleigh formulation with two control frequencies were carried out by employing D-MOD2000 and DEEPSOIL codes. The analyses with simplified formulation with control frequency set at $f_s$ were carried out using only DMOD2000. As mentioned earlier, the accuracy of the formulation was tested by comparing numerical results with those obtained by ProSHAKE. The target damping $\xi^*$ was assumed equal to soil damping ($D = 5\%$) as widely adopted in current practice while the control frequencies were calculated for both codes according to the procedure proposed in QUAD4M code (Table 1) in order to have comparable results between codes.

The numerical results in terms of response spectra at soil deposit surface are summarized for all selected input accelerograms in Figure 4. The results obtained with simplified formulation assuming $f_s$ as control frequency greatly underestimates the reference results by PROSHAKE. This underestimation is particularly severe for motion characterized by predominant frequency well above the fundamental one or, in general, by rich energy content at frequency higher than $f_s$ exciting the vibration mode higher than first (see for example Sturmo NS and Tarcento NS inputs). This behavior is due to the overestimation of damping ratio with respect to the target value in the $f > f_s$ region.

<table>
<thead>
<tr>
<th>Station</th>
<th>Component</th>
<th>Earthquake</th>
<th>Predominant period, $T_p(s)$</th>
<th>Predominant frequency, $f_p(\text{Hz})$</th>
</tr>
</thead>
<tbody>
<tr>
<td>Port Island</td>
<td>NS</td>
<td>Kobe 1994</td>
<td>1.10</td>
<td>0.9</td>
</tr>
<tr>
<td>Taft</td>
<td>S69E</td>
<td>Kern County 1952</td>
<td>0.44</td>
<td>2.3</td>
</tr>
<tr>
<td>Sturmo</td>
<td>NS</td>
<td>Irpinia 1980</td>
<td>0.38</td>
<td>2.6</td>
</tr>
<tr>
<td>Gilroy2</td>
<td>90</td>
<td>Loma Prieta 1989</td>
<td>0.30</td>
<td>3.3</td>
</tr>
<tr>
<td>Gilroy2</td>
<td>50</td>
<td>Coyote Lake 1979</td>
<td>0.18</td>
<td>5.6</td>
</tr>
<tr>
<td>Tarcento</td>
<td>NS</td>
<td>Friuli 1976</td>
<td>0.12</td>
<td>8.3</td>
</tr>
</tbody>
</table>

Table 2 Accelerograms employed for comparative linear analyses on ideal soil profile.

![Figure 2 Soil profile employed for comparative site response analyses](image-url)
The full Rayleigh formulation with two control frequencies generally provides results that are fairly consistent with the results by frequency independent damping analyses for all selected input motions. The maximum differences with ProSHAKE are generally lower than 10% and 20% in time and frequency domains respectively; slightly higher discrepancies can be found only for Tarcento input motion. However, it should be noted that D-MOD2000 results with full Rayleigh formulation systematically overestimate the reference results at all the frequency range. DEEPSOIL analyses with the two control frequencies selected according to the rule implemented in QUAD4M (Hudson et al., 1994) and extended Rayleigh formulation (with four control frequencies) do not show appreciable differences. An exception is the analyses with high frequency Tarcento input motion where the use of extended Rayleigh formulation allows to match ProSHAKE solution much better than the two control frequencies scheme.

4 NONLINEAR ANALYSES

In nonlinear analyses, the use of a true nonlinear constitutive model allows to model the hysteresic behavior of soils. However, Rayleigh damping is added to simulate the energy dissipation at small-strain where nonlinear models provide zero damping.

The Orvieto hill site (Costanzo et al., 2007) was selected to assess the influence of Rayleigh damping formulations in the nonlinear range. The upper part of the Orvieto hill is formed by a 60 m high slab of lithic tuff and weakly cemented pozzolana resting on a overconsolidated clay formation which thickness reaches 200m. The physical and mechanical properties of the materials are reported in Table 3.

1D analyses were carried out at the hill center where 2D effects are negligible at least along the WE direction where the width of the slab is about 1500m. The fundamental frequency of soil deposit is $f_s = 0.61 \text{Hz}$. Similar to the linear case, nonlinear analyses were performed using the same
Figure 4 Linear analyses: comparison of response spectra at soil deposit surface for all six input accelerograms
The soil deposit was excited by a real accelerograms, Cascia NS ($a_{\text{max}} = 0.154\, \text{g}$), representative of near-field ground motion condition (Costanzo et al., 2007). Further, the nonlinear analyses were conducted scaling accelogram to higher input energy, i.e. to 0.5g. The acceleration response spectra (5% structural damping) of the input motions are shown in Figure 5.

The response spectra obtained when excited by both scaled and unscaled accelerograms of the input motion through the soil deposit are shown in Figure 6. Here, case (a) is referred to the analyses using unscaled input motion and case (b) is when scaled accelerograms are used. It is observed that the use of full Rayleigh formulation allows to match ProSHAKE results for both case (a) and (b). Some discrepancies are observed for 0.5g input at periods lower than 0.2s. This behavior could be related to the differences in constitutive models: true nonlinear models are adopted by D-MOD2000/DEEPSOIL (in particular MKZ hyperbolic model) while ProSHAKE adopt a linear equivalent approach. It is worth noting that D-MOD2000 results using full

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**Figure 5, Response spectra of Cascia NS accelerogram**

**Figure 6, Nonlinear analyses: comparison of response spectra at soil deposit surface for unscaled (a) and scaled (b) Cascia NS input motion**
Rayleigh formulation slightly overestimates the results by other codes, in a large part of frequency range, as noted before. On the other hand, like in the linear range, the simplified formulation severely underestimates the response also at high strains (input scaled to 0.5g).

5 CONCLUSIONS

D-MOD2000 and DEEPSOIL 1D computer codes were employed for parametric site response analyses. The aim of the study was the calibration of Rayleigh damping formulations which are implemented in different ways in the two codes. The study was confined on the assessment of the influence of Rayleigh damping options (simplified, full or extended formulation) on site response analyses. Linear analyses carried out on an uniform soil deposit and excited by six different accelerograms has shown that the full and extended Rayleigh damping approximation in DEEPSOIL yield results fairly consistent with the results by reference solution, i.e. frequency independent damping solution (ProSHAKE). On the other hand, for all accelerograms, D-MOD2000 results when full Rayleigh damping is implemented overestimate the reference results and severely underestimate the reference solution when the simplified Rayleigh damping formulation is used. In particular a severe underestimation of ground motion is observed when the predominant frequency of input motion is significantly higher than control frequency. Nonlinear analyses were performed at the Orvieto site and excited through the unscaled and scaled Cascia input motion ($a_{\text{max}}=0.154g$ and $a_{\text{max}}=0.5g$) respectively. When the site is excited through unscaled accelerogram, both DEEPSOIL and D-MOD2000 results with full Rayleigh damping formulation well estimate the reference results. Again the use of simplified Rayleigh damping formulation significantly underestimates the seismic response. On contrary, when the site is excited by scaled accelerogram (high nonlinearity range) there is a significant difference even between DEEPSOIL and reference code especially at medium-high frequencies, mainly because the differences in the adopted constitutive models. Also in highly nonlinear range considerable underestimation can be observed when simplified Rayleigh damping formulation is used.

Based on the analyses presented in the paper, the following practical recommendations can be given: i) the use of simplified formulation should be avoided, even in the nonlinear range, especially for input motion characterized by predominant frequencies well above the control frequency; ii) as shown by Kwok et al. (2007) the use of full formulation with the two control frequencies, chosen according the “QUAD4M procedure”, well match the frequency independent damping solution for practical purposes; iii) the use of extended Rayleigh formulation does not show significant improvement with respect to the full two control frequencies one and should therefore be omitted being time consuming.

6 REFERENCES


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